1 Introduction

Certainly negated predicates (e.g. think, believe, want) invoke a reading where the negation is interpreted in the embedded clause. For example, (1a) implies (1b).

(1) a. John doesn’t think Bill left.
   b. John thinks Bill didn’t leave.

Most other predicates do not trigger such readings. (2a) cannot be interpreted as (2b).

(2) a. John doesn’t claim Bill left.
   b. John claims Bill didn’t leave.

There are two main approaches to Neg-raising (NR):


The syntactic approach (Fillmore, 1963; Horn, 1978; Collins & Postal, 2014)

In this paper we argue that:

- even though the semantic-pragmatic accounts are generally better equipped to account for NR effects, they face substantial challenges too.
- a novel, modified version of the semantic-pragmatic approach can overcome these problems, and can explain the behaviour and distribution of Horn-clauses.
- Horn-clauses, often considered the strongest argument for the syntactic and most problematic for semantic-pragmatic approach, at closer inspection, cannot be explained under syntactic approach but can actually be explained under our novel, modified version of the semantic-pragmatic approach.
2 The pragma-semantic approach and its challenges

- The pragma-semantic approach takes NR readings to be the result of an excluded middle inference. This excluded middle inference is lexically encoded on a particular group of predicates known as Neg-Raising Predicates (NRPs from now on). This approach has two version:
  - NRPs come with an excluded middle presupposition. (Gajewski, 2005, 2007)
  - NRPs have excluded middle alternatives (Romoli, 2012, 2013).

2.1 The presuppositional approach

2.1.1 Outline

- Excluded middle presupposition: The speaker is opinionated about the truth or falsity of the embedded proposition.
- The NR reading is a logical consequence of this presupposition and the literal meaning of the sentence, as shown in (3).

(3) not [NRP [S]]
   Assertion: \( \neg \text{NRP} (S) \)
   Presupposition: \( \text{NRP} (S) \lor \text{NRP} (\neg S) \)
   \( \therefore \text{NRP} (\neg S) \)  
   (Gajewski 2005: p.14).

Let’s apply this account to (4a) to get the NR reading in (4b).

(4) a. John doesn’t think that Bill left.
   b. John thinks that Bill didn’t leave

With the excluded middle presupposition that the speaker thinks that either Bill left or Bill didn’t leave, (4a) entails (4b).

(5) Assertion: It’s not the case that John thinks Bill left. (4a)
    Presupposition: John thinks Bill left \lor John thinks Bill didn’t leave.
    \( \therefore \) John thinks Bill didn’t leave. (4b)

2.1.2 Problems for the presuppositional approach

- The excluded middle doesn’t behave like other presuppositions. For instance, it doesn’t project through conditionals (6a) or questions (6b) (Romoli, 2012; Križ, 2015), and it doesn’t pass the so-called “Hey, wait a minute” test (7) (Križ, 2015).

(6) a. If Mary doesn’t think that Bill should be hired, she will say so at the next faculty meeting.
   b. Does Mary think that Bill should be hired?

(7) a. Mary doesn’t think that Bill should be hired.
   b. #Hey, wait a minute! I didn’t know that she necessarily has an opinion about that.
There are contexts under which NRPs receive a non-NR reading without resulting in a presupposition failure (Homer, 2015).

(8) a. Unlike many people nowadays, my great-grandparents didn’t want to spend a lot of time on the internet.
    b. My great-grandparents wanted not to spend all their spare time on the internet.

(9) At a job interview.
    a. I don’t want to make a lot of money, you know.
    b. I want not to make a lot of money.

In many contexts, the universal (or even existential) projection of an excluded middle presupposition from the scope of negative indefinites is too strong.

(10) *It’s the first day of school; before entering the (new) school your mom tells you:
    a. Remember, nobody here thinks you’re stupid.
    b. Everybody here thinks you’re not stupid.
    c. Somebody here thinks you’re not stupid.

For the NR reading to be true, not only everybody should have an acquaintance relation with you but also have an opinion about whether or not you’re stupid.

In certain contexts, certain non-NRPs, like non-factive *know, dubbed cloud of un-knowing*-predicates, nevertheless get a NR reading, as illustrated below.

(11) Trump: I can overturn the result of the election.
Constitutional lawyer: I don’t know/ am not sure that’s constitutionally possible, sir.

(12) a. Anthony: you know why?
    b. Uncle Junior: I don’t know that I give a f***.

The examples below from Horn (2014) show that such predicates can also license strict NPIs in their complement, a general footprint of NR.

(13) a. I don’t know that Santa comes around these parts until Christmas Eve.
    b. I can’t say I’ve cooked myself a full meal in weeks, if not months.

2.2 The implicature approach

2.2.1 Outline

To circumvent problems concerning the presuppositional account of NR, Romoli (2012, 2013) proposes a scalar implicature account of NR.

Instead of assuming the excluded middle statement is a presupposition, Romoli (2012, 2013) derives it as an implicature.
NRPs have the excluded middle statement as a lexical alternative.

(14)  \( \text{Alt}(\text{think } p(\mathbf{x})) = \{ \text{think}_x p, \text{think}_x p \vee \text{think}_x \neg p \} = \{ \Box_x p, \Box_x p \vee \Box_x \neg p \} \)

(15)  
\begin{align*}
&\text{a. John doesn’t believe that it is raining.} \\
&\text{b. } \neg \text{believe}_j p
\end{align*}

(16)  
\begin{align*}
&\text{a. } \text{Alt}(\neg \text{believe}_j p) = \{ \neg \text{believe}_j p, \neg(\text{believe}_j p \vee \text{believe}_j \neg p) \} \\
&\text{b. } \exists \text{EXH}(\neg \text{believe}_j p) = \neg \text{believe}_j p \wedge \neg \neg(\text{believe}_x p \vee \text{believe}_j \neg p) = \neg \text{believe}_j p \wedge (\text{believe}_j p \vee \text{believe}_j \neg p) \\
&\text{c. } \text{believe}_j \neg p
\end{align*}

Any scalar implicature account of NR has the advantage of not running into the projection problems of the presuppositional account (as mentioned in (1)).

as the generation of scalar implicatures depends on the contextual relevance of particular alternatives, the problem addressed in (2) doesn’t arise either.

2.2.2 Problems for the implicature approach

Romoli’s special implementation at the same time relies on two unmotivated assumptions:

- The implicature calculation is based on the assumption that NRPs have excluded middle statement as a ‘lexical’ alternative. But this lexical alternative is hardly pronounceable and is not attested elsewhere. (Križ, 2015)
- Romoli postulates that universal quantifiers (over possible worlds) in natural language must either have an existential or an excluded middle alternative. This choice, however, is only taken as a matter of conventional properties of lexical items and doesn’t follow from any general pragmatic principles.
- Romoli’s account cannot solve problem (3).

3 A novel approach

We propose a new implementation of scalar implicature account that can solve problems (1)-(3), without the need for unmotivated assumptions like Romoli’s.

Our analysis has two components:

- (Strict) duality: \( \neg \forall \Leftrightarrow \exists \neg \) under presupposition preservation
- Strengthening of subdomain alternatives (Chierchia, 2013)

The Exhaustivity operator can apply to a strict logical equivalences of an LF.

We follow Buccola et al. (2021) in taking alternatives being objects at level of LF, and not necessarily linguistic objects (words/phrases).

We furthermore assume that a strictly equivalent LF can be used as a basic meaning for implicature calculation, only if the universal modal lacks an existential dual in the lexicon (Deal, 2011; Jeretič, 2021)).
3.1 Duality

- The literal meaning of negated NRPs is equivalent to: $\exists w \in W: \neg p(w)$.
  
  (by strict duality: $\neg \forall w \in W: p(w) \iff \exists w \in W: \neg p(w)$)

- This weak existential reading can be further subject to strengthening.

- Logical equivalence in a trivalent system, where the possible truth-values are $\{1,0,#\}$ and presupposition failure is marked by the third truth-value, is defined as follows:

  (17) **Strict equivalence**

  \[ p \iff_{strict} q \text{ iff } p \implies_{strict} \text{ entails } q \text{ and } q \implies_{strict} \text{ entails } p \]

  (18) **$\Rightarrow_{strict}$ entailment**

  \[ p \Rightarrow_{strict} \text{ entails } q \text{ iff } \forall w : [p](w) = 1 \implies [q](w) = 1 \]

- Duality is presupposition-preserving.

- Consequently, Strict duality does not hold for all modals.

- Modals might carry presuppositions that block duality: e.g. factive know:

  Assume $\diamond K_p$ is the existential dual knowledge operator of $\Box K_p$.

  - If the existential knowledge operator also carries the factivity presupposition that the embedded $p$ is true, the dual rule is not valid.

    (19) $p = 1. \neg \Box p \iff_{strict} \neg p = 1. \Diamond \neg p$

    Even when $\diamond K_p$ doesn’t carry any presupposition, the strict duality is still not valid.

    In a world where the factivity presupposition is not satisfied, $\neg \Box p(w)$ is $\#$ but $\Diamond \neg p(w)$ is true.

    (20) $p = 1. \neg \Box p \iff_{strict} \Diamond \neg p$

- Since Strict duality does not hold for these predicates, no weak existential reading can be derived that can, in turn, be further strengthened

  $\rightarrow$ **strictly non-NRPs**, like factives, never yield NR readings

- This means that is not NRPs that are special in allowing NR inferences; it is rather strictly non-NRPs that are special in not allowing them.

3.2 Exhaustification

- Parallel to the implicature account of Free Choice (Fox, 2007; Bar-Lev & Fox, 2017), and Homogeneity (Bassi & Bar-Lev, 2018; Magri, 2014; Bar-Lev, 2020), we take strengthened readings to be the result of the application of an exhaustivity operator at LF.

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1We thank Amir Anvari for pointing to us that using the definition of Strawson entailment would not provide us with the correct result.
The exhaustification triggers subdomain alternatives (Bassi & Bar-Lev, 2018; Jeretič, 2021; Staniszewski, 2021).

Subdomain alternatives of existential quantification over a plurality are not closed under conjunction. The conjunction of the sub-domain alternatives is not a member of \( \text{Alt}(\exists w \in \{w_1, w_2, w_3\}: \neg p(w)) \).

We adopt the definition of the exhaustivity operator (EXH) by Bar-Lev & Fox (2017).

(21) Innocent Exclusion + Innocent Inclusion–based exhaustivity operator:
\[
\text{EXH}^{IE+II}(C)(p)(w) \iff \forall q \in \text{IE}(p, C)[\neg q(w) \land \forall \in \text{II}(p, C)[r(w)]
\]

(22) Given a sentence \( p \) and a set of alternatives \( C \):
   a. \( \text{IE}(p, C) = \bigcap\{C' \subseteq C : C' \text{ is a maximal subset of } C, \text{ s.t.} \{\neg q : q \in C'\} \cup \{p\} \text{ is consistent}\} \)
   b. \( \text{II}(p, C) = \bigcap\{C'' \subseteq C : C'' \text{ is a maximal subset of } C, \text{ s.t.} \{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in \text{IE}(p, C)\} \text{ is consistent}\} \)

EXH takes a proposition (\( p \)), and a set of alternatives (\( C \)) as arguments, and returns the conjunction of all of the negated innocently excludable (IE) alternatives, and all of the asserted innocently includable (II) alternatives.

The IE alternatives are all those that can be assigned false consistently with the prejacent.

The II alternatives are those that can be assigned true consistently with the prejacent and the falsity of all IE alternatives.

The NR reading is then derived via application of EXH, starting with the LF corresponding to the basic weak reading (\( \exists w \in W: \neg p(w) \))

Let’s assume the speaker’s belief worlds consists of three worlds \( w_1, w_2 \) and \( w_3 \).

The alternatives generated from replacing the domain variable with its subsets in the weak, existential reading are given in (23).

(23) \( \exists w \in \{w_1, w_2, w_3\}: \neg p(w) \), \( \exists w \in \{w_1, w_2\}: \neg p(w) \), \( \exists w \in \{w_1, w_3\}: \neg p(w) \), \( \exists w \in \{w_2, w_3\}: \neg p(w) \), \( \exists w \in \{w_1\}: \neg p(w) \), \( \exists w \in \{w_2\}: \neg p(w) \), \( \exists w \in \{w_3\}: \neg p(w) \)

No alternatives are IE.

All alternatives are II.

Upon exhaustification, we will have (24), which is equivalent to the NR reading.

(24) \( \text{EXH}^{IE+II}(\text{Alt}(\exists w \in \{w_1, w_2, w_3\}: \neg p(w)))(\exists w \in \{w_1, w_2, w_3\}: \neg p(w)) = \exists w \in \{w_1, w_2, w_3\}: \neg p(w) \land \exists w \in \{w_1, w_2\}: \neg p(w) \land \exists w \in \{w_1, w_3\}: \neg p(w) \land \exists w \in \{w_2, w_3\}: \neg p(w) \land \exists w \in \{w_1\}: \neg p(w) \land \exists w \in \{w_2\}: \neg p(w) \land \exists w \in \{w_3\}: \neg p(w) \equiv \forall w \in \{w_1, w_2, w_3\}: \neg p(w) \)
3.3 Weak readings

There are certain contexts where the NR reading does not arise, as in (25).

(25) *It’s the first day of school, before entering the school your mom tells you: Remember, nobody here thinks you’re stupid.*

Following Bar-Lev’s (2018; 2020) account of non-maximal readings of definite plurals, we take the non-NR reading to be the result of pruning all the subdomain alternatives which involve singleton sets (i.e. \{w_1\}, \{w_2\}, \{w_3\}).

By applying EXH to this set of alternatives, we get the weak non-NR reading.

(26) a. \(\exists w \in \{w_1, w_2, w_3\} \) : \(\neg p(w)\), \(\exists w \in \{w_1, w_2\} : \neg p(w)\), \(\exists w \in \{w_1, w_3\} : \neg p(w)\), \(\exists w \in \{w_2, w_3\} : \neg p(w)\)

b. \(\text{EXH}^{IE+II}(\text{Alt}(\exists w \in \{w_1, w_2, w_3\} : \neg p(w)))\ (\exists w \in \{w_1, w_2, w_3\} : \neg p(w)) = \exists w \in \{w_1, w_2, w_3\} : \neg p(w)\)

Under this view, the (un)availability of strengthened (NR) readings for duality-allowing epistemic modals depends on which set of alternatives EXH applies over.

When EXH applies over the whole set of subdomain alternatives, we get the strengthened reading.

When EXH applies over the subset remained after pruning singleton sets, we get the weak reading.

Pruning is a mechanism to reduce the set of alternatives to only those that are plausible and relevant in a given context.

(27) a. **Maxim of Relevance:** Every utterance must be relevant to Q.

b. **Weakening:** Pruning can only weaken the meaning. (Crnič et al., 2015)

c. **Minimal pruning:** Don’t prune more than necessary to satisfy.

(Bar-Lev, 2020)

Singleton set alternatives are normally pruned when modals express objectivity or evidentiality, because access to facts in a possible world is implausible.

→ In such contexts, (NR) readings are predicted to be impossible.
4 Advantages

4.1 Neg-raising under DE

- An argument for the implicature account of NR comes from cases where negated NRPs are embedded in a downward-entailing (DE) environment.

- As is well known, implicatures cannot be embedded under a DE operator, unless the relevant scalar term bears pitch accent (Fox & Spector, 2018; Horn, 1989).

- This prediction seems to be borne out. As shown in (28)-(29), the strengthened NR reading can only be generated with a pitch accent on “don’t”. In the absence of such marked pronunciation, the strong NPI in years is not licensed as NR reading is required for licensing strict NPIs in an embedded clause.

- As predicted, no strengthened NR reading can then be generated, as shown in (28), where the absence of the anti-additive NR context over the embedded clause results in the strong NPI in years being unlicensed.

(28)  a. *Few people don’t think Sue has visited in years.
       \[\text{\textlangle} \] Few people think Sue has not visited in years.
       b. *At most three people don’t think Sue has visited in years.
       \[\text{\textlangle} \] at most three people think Sue has not visited in years.
       c. *I doubt that Mary doesn’t think Sue has visited in years.
       \[\text{\textlangle} \] I doubt that Mary thinks Sue has not visited in years.

(29)  a. Few people DON’T think Sue has visited in years.
       \[\text{\textlangle} \] Few people think Sue has not visited in years.
       b. At most three people DON’T think Sue has visited in years.
       \[\text{\textlangle} \] at most three people think Sue has not visited in years.
       c. I doubt that Mary DOESN’T think Sue has visited in years.
       \[\text{\textlangle} \] I doubt that Mary thinks Sue has not visited in years.

4.2 Neg-raising is not a lexical property

- The (novel) observation that (11), repeated below, has a NR reading, even though non-factive know doesn’t always give rise to them, shows that the ability to trigger a NR reading must not be a lexical property of predicates.

(30)  Trump: I can overturn the result of the election.
        Constitutional lawyer: I don’t know/ am not sure that’s constitutionally possible, sir.

- Our approach to NR is the only approach that can account for this observation

- all other theories of NR, including Križ’s suggestion to take NRPs as involving not universal quantification over worlds, but homogeneous distributive predication over a plurality of worlds, take NRPs to be a special class of verbs with some unique lexically-encoded property enabling them to yield NR readings.
Since the calculation of implicatures is context-dependent, we predict that every negated universal modal whose presuppositions do not block duality, like non-factive know and be sure in (11), can get a NR-reading, provided that the whole set of subdomain alternatives is contextually relevant.

5 The syntactic approach to NEG RAISING and its challenges: the strength of the *Horn clause argument*

5.1 The syntactic Account

Negation is base-generated in the embedded clause and then raises to the higher clause via syntactic movement (Fillmore 1963, Horn 1971 and Collins & Postal 2014).

The lowest copy of NEG is semantically interpreted and the highest copy of NEG is phonologically realized. The syntactic structure of (1a) would then be as in (31).

(31) John NEG think Bill <NEG> left.

The classic syntactic approach has been challenged on several grounds.

For one, NR involving negative indefinites cannot be accounted for in terms of simple semantic reconstruction.

(32) Nobody thinks nuclear war is winnable.

(32) lacks the reconstructed reading of (33):

(33) NEG somebody thinks nuclear war is <NEG> winnable.

(32) means that everybody thinks that nuclear war is not winnable, not that somebody thinks it’s not, a reading that cannot be straightforwardly derived along the lines of the syntactic approach.

For many other problems for the syntactic approach, see a.o. (Romoli, 2012, 2013; Zeijlstra, 2018).

5.2 *Horn*-clauses

Despite these problems, Collins & Postal (2014) present one major argument that has not been countered in the literature so far, see (Romoli, 2012, 2013; Zeijlstra, 2018; Crowley, 2019): *Horn*-clauses.

*Horn*-clauses are instances where subject-auxiliary inversion is licensed not by a negative quantifier in SPEC,CP, but rather by an NPI in SPEC,CP, which in turn is licensed by a negated NR-predicate, as in (34).

(34) [I don’t think that [anywhere did he mention my book]]
Since Negative Inversion (subject-auxiliary inversion under negation) applies in a strictly local fashion, Collins & Postal (2014) take the existence of Horn-clauses as strong evidence for a syntactic approach to Neg-raising.

Only under a local approach can the negation in the main clause have appeared in SPEC,CP at an earlier stage of the derivation, as in (35), where < … > denotes a lower copy.

(35) I do NEG think that [<NEG> anywhere] did he mention my book [<NEG anywhere>]

Nevertheless, Collins & Postal’s analysis suffers from at least three major problems.

1. It cannot exclude universal quantifiers from appearing in Horn-clauses. Whereas (36a) is fully acceptable, (36b) is not. The structure in (36c) that derives (36b) should, in principle, be possible in Collins & Postal’s system.

(36) a. Not everywhere did he mention my book
    b. *I don’t think that everywhere did he mention my book
    c. *I do NEG think that [<NEG> everywhere] did he mention my book [<NEG everywhere>]

The only solution that Collins & Postal offer is to rule out (36c) by postulate a condition that bans negated non-existentials from triggering Horn-clauses (cf. Collins & Postal (2014)).

2. The second problem for Collins & Postal (2014) is that the set of negative predicates that can license Horn-clauses is not restricted to negated NR-predicates. Horn (2014) points out that non-factive know, be aware, and some other predicates, which he dubs Cloud of Unknowing predicates, license Horn-clauses as well, as shown in (37).

(37) I *(don’t) know that ever before had all three boys napped simultaneously

But in (37), there is no semantic reflection of negation in the embedded clause, i.e. (37) lacks a NR-reading.

To resolve this, Collins & Postal (2018) stipulate that Cloud of Unknowing-predicates cannot be outscoped by a raised negation. This would then rule out the NR-reading of (37).

For them, the underlying structure of (37) must contain two additional negations, one of which is raised into the matrix clause, and both of them being phonologically deleted:

(38)[I do NEG₁ know NEG₂ [<NEG₂ > that NEG₃ ever before had all three boys napped simultaneously]]

However, apart from such an escape hatch being purely stipulative, Collins & Postal (2018) predict that (39) should still be fine with a NR-reading (40), contrary to fact.²

(39) Nobody doesn’t know that ever before had all three boys napped simultaneously].

²This system of multiple (deleted) negation is also what according to Collins & Postal (2014) is behind examples like (32):

(i) [Nobody NEG₁ supposes NEG₂ [that <NEG₂ > nuclear war NEG₃ is winnable]]
(40) [Nobody NEG₁ know [that <NEG₁> ever before had all three boys napped simultaneously]]

Finally, the same problem as with Cloud of Unknowing-predicates arises with many other predicates, for instance with accept. Crucially, these predicates are not NR-predicates, but when negated, they still can license subject-auxiliary inversion with an NPI in SPEC,CP, ((41)).

(41) I *(didn’t) accept that any of those problems had she ever really solved

For (41), Collins & Postal (2014) argue that here the NPI any of those problems takes matrix scope and therefore, examples like (41) are different from real Horn-clauses.

However, the claim that any of those problems in (41) takes matrix scope is false. If it were the case, (41) should be felicitous in a scenario where we know that Mary solved some problems, but we don’t know which ones (e.g., when solving some problems is a requirement for passing a test, and we only know that Mary passed the test).

In sum, Collins & Postal’s proposal suffers from several insurmountable problems.

At the same time, as of date, no existing alternative account for Horn-clauses has been proposed that does not require movement of negation.

Our novel approach can actually account for the overall distribution and readings of Horn-clauses.

5.3 Horn-clauses

As a starter, Negative Inversion is not obligatory in syntax, but an optional instance of movement that triggers particular semantic effects (cf. (Büring, 2004)).

(42) a. With no job is Kim happy
   b. With no job Kim is happy

This suggests that Negative Inversion involves LF licensing, and not negative feature-driven syntactic movement.

T-C movement followed up by fronting a negative phrase is fine as long as the clause will receive a sentential negation reading, i.e., if the existential quantifier binding the event variable ends up in an Anti-Additive context.

Under a duality-based approach, any universal predicate that allows duality, gives rise to an equivalent LF where negation scopes below this predicate.

This includes both NRPs and Cloud of Unknowing predicates like non-factive know, but also predicates such as accept.
All these predicates allow duality to apply, even if the dual readings are not further strengthened into \(NR\) readings.

This is in line with Horn’s 2014 conclusion that the possibility or likelihood of \(\neg p\) is an important factor in licensing Horn-clauses – due to duality, \(\neg p\) is evaluated.

The crucial factor in licensing embedded subject-aux ("Horn clauses") and strict NPIs when these occur under higher negation in the sequence \([ a \text{ NEG-Fs that } p]\) is not the requirement that \(F\) be a \(NR\) predicate perse but the existence of a robust association between \(a\) being in a \(\text{NEG-F relation to } p\) and \(a\) being in an \(F'\) relation to \(p\) where \(F'=F\) or \(F'<F\) on a relevant scale.

Such an LF introduces an Anti-Additive context that, unless this Anti-Additivity is disrupted, involves the embedded clause as well.

Given duality, negated (\(NR\)-)predicates should thus allow subject-auxiliary inversion, as long as scopally nothing intervenes between the negation and lower \(C'\) that disrupts Anti-Additivity.

At the same time, the question remains open as to why Horn-clauses require the presence of an NPI in the embedded clause. Why would (36b), repeated below, be out?

(43) *I don’t think that everywhere did he mention my book

The reason lies in the requirements for Negative Inversion / subject-auxiliary inversion itself.

Since \(C'\) must end up in an Anti-Additive context, Negative Inversion is only allowed if the material in embedded \(\text{SPEC,CP}\) is Anti-Additive (or at least Strawson Downward Entailing, cf. (Büring, 2004)) as well.

This, essentially means that duality can license subject-auxiliary inversion, if the material in \(\text{SPEC,CP}\) does not disrupt Anti-Additive either. In (43), the relevant LF would contain \textit{everywhere}, which breaks this criterion.

In order not to being able to disrupt Anti-Additivity the non-negative material in \(\text{SPEC,CP}\), must consist of:

- an existential/indefinite that
- does not give rise to any non-Anti-Additive inferences itself.

As the reader can check in (44), this is only the case for negated NPIs. Other existentials/indefinites give rise to specificity effects or existential import of some sort.

(44)  a. Not anywhere did she go \(\longrightarrow\) Nowhere did she go.
    b.*Not somewhere did she go. \(\longrightarrow\) Nowhere did she go.
    c.*Not to a place in Franc did she go. \(\longrightarrow\) Nowhere in France did she go to.

Consequently, to ensure that the embedded \(C'\) ends up in an Anti-Additive environment, every \textit{Horn}-clause must contain an NPI in its embedded \(\text{SPEC,CP}\).
This explains the full pattern of *Horn*-clauses without alluding to syntactic movement.

*Horn*-clauses also provide another argument for the necessity of duality step.

### 6 Conclusions

- There are contexts under which NRPs receive a non-NR reading without resulting in a presupposition failure.
- In certain contexts, some non-NRPs (e.g. non-factive know) can get a NR reading.
  
  → the ability to trigger a NR reading is *not* a lexical property of predicates.
- We propose a new implementation of scalar implicature account that solves these problems. Our analysis has two components:
  - **Strict duality**: \( \neg \forall \iff \exists \neg \) under presupposition preservation
  - **Strengthening of subdomain alternatives**
- We predict that negated universal modals whose presuppositions don’t block duality *can* get a NR reading.
- The (un)availability of NR readings for duality-allowing modals is reduced to whether EXH applies over the whole set of subdomain alternatives (strengthened reading) or over a subset after pruning singleton sets (weak reading).
- A non-NR reading is the result of pruning the subdomain alternatives which are singleton sets.
- Given Strict duality the existence of *Horn*-clauses receives a natural explanation, rendering a (final) strong argument in favour of the syntactic approach to NR obsolete.

### 7 Appendix

#### 7.1 Duality is a necessary step

- The question arises as to whether the duality is necessary, as applying EXH directly to \( \neg \Box \) is shown to yield a strong \( \Box \neg \) reading by Jeretič (2021), who provides an implicature account of the scopal interaction between negation and French necessity modals *falloir* and *devoir*.

- The wide scope interpretations of these modals over negation are accounted for via obligatory double strengthening from a \( \neg \Box \) to \( \Box \neg \), as shown below:

\[
\begin{align*}
(45) & \quad S = \neg \Box_{\{w_1,w_2\}} p \\
& \quad \text{Alt}(S) = \{ \neg \Box_{\{w_1,w_2\}} p, \neg \Box_{\{w_1\}} p, \neg \Box_{\{w_2\}} p \} \\
& \quad S' = \text{EXH[Alt(S)]}[S] = \neg \Box_{\{w_1,w_2\}} p
\end{align*}
\]
Jeretič (2021) provides an implicature account of the scopal interaction between negation and French necessity modals *falloir* and *devoir*.

We think Strict duality is a necessary step in strengthening, as direct strengthening of $\neg \square$ runs into following problems:

- The first application of EXH does not have any semantic effect.
- There are alternatives containing universal quantification over singleton sets. We find this problematic. For implicature calculation, it is important to be able to assign True or False to alternatives. However, it is hard to assign a truth value False or True to sentences with universal quantification over singleton sets.

(47) a. #Every current Pope is Italian.
    b. #Not every current Pope is Italian.

Moreover, since $\neg \square_{\{w_1\}} p$ is true in case $w_1$ is a $\neg p$ world, $\neg \square_{\{w_1\}} p$ will end up being equivalent to $\neg \diamond_{\{w_1\}} p$. Similarly, $\square_{\{w_1\}} p$ will be equivalent to $\diamond_{\{w_1\}} p$. Therefore, allowing universal quantification over singleton sets blurs the distinction between universal and existential quantifiers.

7.2 NR as the literal meaning (Staniszewski, 2021)

- Weak necessity modals like *should* and *supposed to* are existential quantifiers over possible worlds.
- In UE environments, this weak existential reading is strengthened to $\forall$ via $\text{EXH}^{IE+II}$
- There is no strengthening under negation $\Rightarrow \neg \exists$ (NR reading)
- There are several problems with this account:
  - To be extendable to the phenomenon of NR broadly, *believe* and *think* also need to be treated as existential quantifiers over possible worlds.
  - It cannot account for the observation that *cloud of unknowing* predicates can also give rise to NR readings.
  - It cannot account for the observation in (28) that NR readings are not generated when negated NRPs are further embedded under DE.
  - Non-NR readings (as shown in (8)-(10)) are counterintuitively derived as cases of ”strengthening” under negation.
References


