Three Improvements to the HPSG Model Theory

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Coda O

Problems with HPSG model theory

Aim: propose three orthogonal improvements to the standard HPSG model theory (RSRL; Richter 2004)

- solve the long-standing "Höhle's problem" of spurious ambiguities,
- solve various problems stemming from the insistence on **exhaustive models**,
- propose a **second-order extension** to deal with the analysis of unlike category coordination in Yatabe 2004.

Exhaustive models

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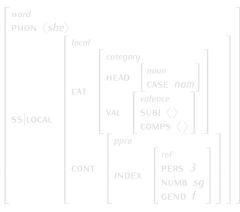
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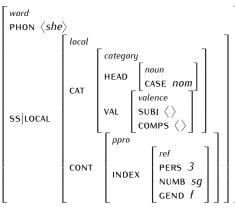
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- $2 \times she$ (assume different *refs*):



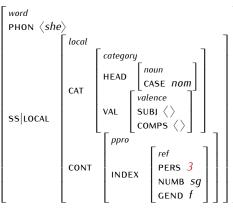
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- assuming there is just one elist, there are 168 different configurations for 2 × she!
- 5208, taking into account verbal HEAD values,
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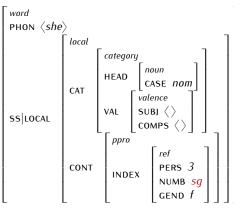
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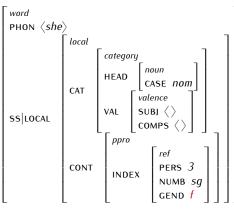
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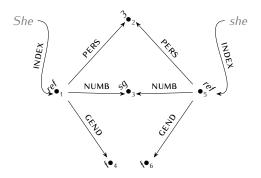
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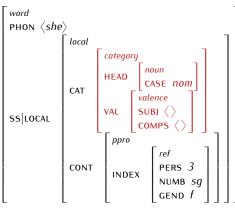


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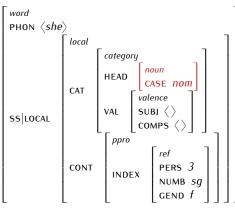
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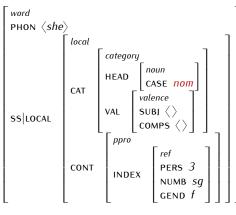
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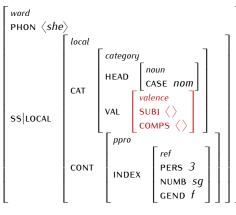
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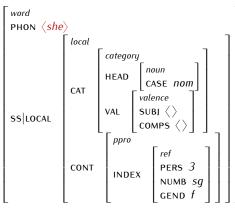
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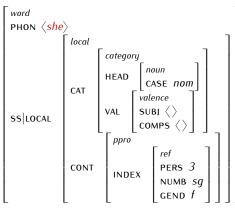
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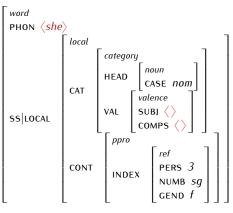
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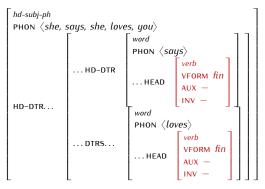


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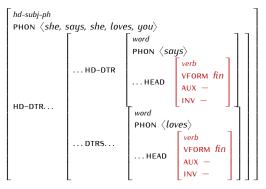


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Partial s	olutions			

Unique Empty List Condition (Richter 2007: 102): • $\forall \mathbb{I} \forall \mathbb{I} ((\mathbb{I} \sim elist \land \mathbb{I} \sim elist) \rightarrow \mathbb{I} \approx \mathbb{I})$

- $\forall 1 \forall 2 (are-copies(1, 2) \rightarrow 1 \approx 2)$
- are-copies(1,2) informally (cf. Sailer 2003:116, (130)):
 - 1 and 2 are of the same species, and
 - values of corresponding attributes of 1 and 2 stand in the are-copies relation.
- In Sailer 2003, the scope of GIP is constrained to configurations encoding Ty2 semantic formulae.

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- **otherwise**, reliance on the non-identity of indiscernibles **very rare** in HPSG, but not unheard of:
 - Höhle 1999: §2.4 assumes that some "identities are not token but type identities",
 - Meurers 1998: 326, fn. 42 assumes "that the HEAD values of different head projections are never (accidentally) token identical, which could be explicitly enforced by a constraint on unembedded signs."

- make GIP universal by default,
- but allow grammars to specify that **objects of certain sorts** (e.g., *ref*) **escape GIP**.

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Technicalities:

- If objects of sort σ are meant to be intensional, then σ is specified for **the attribute** INT **(intensional)** with values of sort σ .
- Universal Intensionality Principle: ∀1∀2 (1[INT 2] → 1 ≈ 2)

For **example**, INDEX values of 2 × *she*:



- Does GIP enforce $1 \approx 2$? (recall GIP: $\forall 1 \forall 2 (are-copies(1,2) \rightarrow 1 \approx 2)$)
- That is, are 1 and 2 copies?
- are-copies(1,2) iff 1) same species (yes), and 2) values of PERS are copies (yes), and 3) NUM (yes), and 4) GEND (yes), and 5) values of INT.
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	pers 3			pers 3	
1	NUM <i>sg</i>	and	2	NUM <i>sg</i>	
	gend f			gend f	
	INT 1			INT 2	

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Introduction O	Höhle's problem ○○○●○○○○	Exhaustive models	Second-order extension	Coda O
Technicalities				

- If objects of sort σ are meant to be intensional, then σ is specified for **the attribute** INT **(intensional)** with values of sort σ .
- Universal Intensionality Principle: $\forall 1 \forall 2 (1 \text{ [INT 2]} \rightarrow 1 \approx 2)$

	ref			ref	
	pers 3			pers 3	
[]	NUM <i>sg</i>	and	2	NUM sg	
	gend f	-		gend f	
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Technicalities				

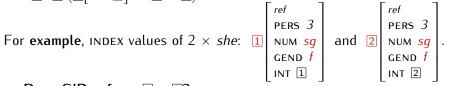
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- If objects of sort σ are meant to be intensional, then σ is specified for the attribute INT (intensional) with values of sort σ .
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	ref			ref	
	pers 3			pers 3	
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Introduction O Höhle's problem

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Scope of the solution

This is a **complete solution** if the grammar does not produce any other cyclic structures.

- the grammar writer deals with that (aka 'not my problem'),
- a more complex solution see the extended slides.

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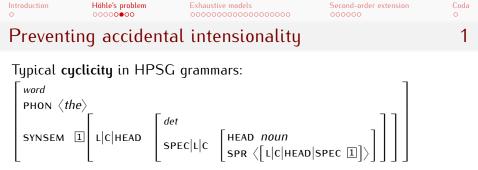
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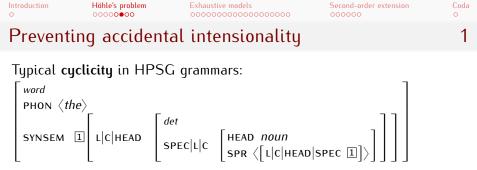
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In the following, assume the two NPs *the dog* have the same INDEX value: • *A man with the dog likes the dog.*

Then there are **two analyses**, with the indiscernible NPs identical or not. **A possible** (ugly) **solution**:

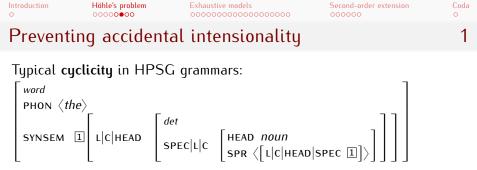
- if two *different* phrases 1 and 2 (here: *with the dog* and *likes the dog*)
- have on their DTRS lists elements 1d and 2d (here: 2 × the dog),
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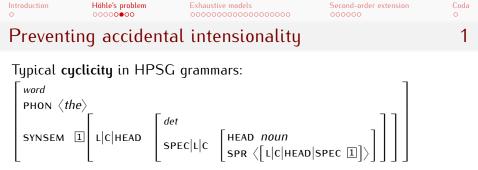


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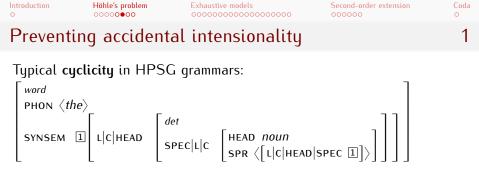


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Introduction O	Höhle's problem	Exhaustive models	Second-order extension	Coda O
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Sailer 2003: 116:

$$\begin{array}{l} \forall \texttt{1} \forall \texttt{2} \; \texttt{are-copies}(\texttt{1},\texttt{2}) \leftrightarrow \\ \left(\bigvee_{\sigma \in \mathcal{S}} (\texttt{1} \sim \sigma \land \texttt{2} \sim \sigma) \land \bigwedge_{\alpha \in \mathcal{A}} (\texttt{1} \alpha \approx \texttt{1} \alpha \rightarrow \texttt{are-copies}(\texttt{1} \alpha,\texttt{2} \alpha)) \right) \end{array}$$

Idea: recursion with memory, immune to cycles - nothing is intensional:

 $\forall \texttt{1} \forall \texttt{2} \texttt{ are-copies}(\texttt{1},\texttt{2}) \leftrightarrow \texttt{Rcopies}(\langle \rangle,\texttt{1},\langle \rangle,\texttt{2})$

 $\begin{array}{l} \forall \texttt{I} \forall \texttt{2} \; \texttt{Rcopies}(\texttt{I},\texttt{I},\texttt{I},\texttt{I}\texttt{2},\texttt{2}) \leftrightarrow \\ \texttt{member2}(\texttt{I},\texttt{L}\texttt{1},\texttt{2},\texttt{L}) \lor \\ \end{array}$

$$\left(\bigvee_{\sigma\in\mathcal{S}}(\mathbb{1}\sim\sigma\wedge\mathbb{2}\sim\sigma)\wedge\bigwedge_{\alpha\in\mathcal{A}}(\mathbb{1}\alpha\approx\mathbb{1}\alpha\to\operatorname{Rcopies}(\langle\mathbb{1}|\mathbb{1}\rangle,\mathbb{1}\alpha,\langle\mathbb{2}|\mathbb{1}2\rangle,\mathbb{2}\alpha)\right)$$

 $\begin{array}{l} \texttt{member2(1), \langle \texttt{L1}h | \texttt{L1}t \rangle, \texttt{2}, \langle \texttt{L2}h | \texttt{L2}t \rangle \rangle \leftrightarrow \\ (\texttt{1} \approx \texttt{L1}h \land \texttt{2} \approx \texttt{L2}h) \lor \texttt{member2(1), \texttt{L1}t, \texttt{2}, \texttt{L2}t \end{array}$

Introduction	Höhle's problem	Exhaustive models	Second-order extension	Coda
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Introduction O	Höhle's problem ○○○○○○●○	Exhaustive models	Second-order extension	Coda O
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Introduction	Höhle's problem	Exhaustive models	Second-order extension	Coda
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$$\left(\bigvee_{\sigma\in\mathcal{S}}(\mathbb{1}\sim\sigma\wedge\mathbb{2}\sim\sigma)\wedge\bigwedge_{\alpha\in\mathcal{A}}(\mathbb{1}\alpha\approx\mathbb{1}\alpha\to\operatorname{Rcopies}(\langle\mathbb{1}|\mathbb{L}1\rangle,\mathbb{1}\alpha,\langle\mathbb{2}|\mathbb{2}\rangle,\mathbb{2}\alpha))\right)$$

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Introduction	Höhle's problem	Exhaustive models	Second-order extension 000000	Coda
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Preventing accidental intensionality				2

Sailer 2003: 116:

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$$\forall \exists \forall \exists \text{Rcopies}(\underline{\texttt{L}}, \underline{\texttt{I}}, \underline{\texttt{L}}, \underline{\texttt{Z}}) \leftrightarrow \\ \text{member2}(\underline{\texttt{I}}, \underline{\texttt{L}}, \underline{\texttt{Z}}, \underline{\texttt{L}}) \lor \\ \checkmark$$

$$\left(\bigvee_{\sigma\in\mathcal{S}}(\mathbb{1}\sim\sigma\wedge\mathbb{2}\sim\sigma)\wedge\bigwedge_{\alpha\in\mathcal{A}}(\mathbb{1}\alpha\approx\mathbb{1}\alpha\to\operatorname{Rcopies}(\langle\mathbb{1}|\mathbb{1}\rangle,\mathbb{1}\alpha,\langle\mathbb{2}|\mathbb{1}2\rangle,\mathbb{2}\alpha))\right)$$

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Now we need to make some structures intensional again:



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```

 $\forall \texttt{1} \mathsf{int}(\texttt{1}) \leftrightarrow (\texttt{1} \sim ref \lor \dots)$



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$$\begin{split} \forall \mathbb{I} \forall \mathbb{2} \; & \operatorname{Rcopies}(\mathbb{I}, \mathbb{I}, \mathbb{I}, \mathbb{2}, \mathbb{2}) \leftrightarrow \\ & \operatorname{member2}(\mathbb{I}, \mathbb{I}, \mathbb{2}, \mathbb{I}) \lor \\ & \bigvee_{\sigma \in \mathcal{S}} (\mathbb{I} \sim \sigma \land \mathbb{2} \sim \sigma) \land \\ & \bigwedge_{\alpha \in \mathcal{A}} (\mathbb{I} \alpha \approx \mathbb{I} \alpha \rightarrow \operatorname{Rcopies}(\langle \mathbb{I} | \mathbb{I} \mathbb{I} \rangle, \mathbb{I} \alpha, \langle \mathbb{2} | \mathbb{I} \mathbb{2} \rangle, \mathbb{2} \alpha)) \land \\ & \mathbb{I} \sim ref \rightarrow \operatorname{Rcopies}(\mathbb{I} \mathbb{I}, \mathbb{I}, \mathbb{I} \mathbb{2}, \mathbb{2}) \end{split}$$



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```

```
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```

King 1999 (and Richter 2004, 2007): (R)SRL models are exhaustive:

- each licensed configuration must actually be present in the model,
- intuition: (R)SRL models are **models of whole languages**, not models of single utterances.

This idea leads to **multiple problems** (cf. Richter 2007):

- twin structures: shared by different utterances,
- (monster) stranded structures: not parts of utterances,
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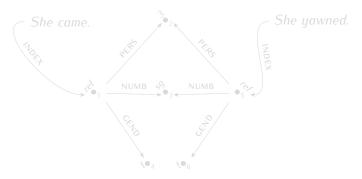
Höhle's problem 00000000 Exhaustive models

Second-order extension

Problem: Twin structures

Richter 2007: nothing in (R)SRL prevents configurations in which different utterances share structure.

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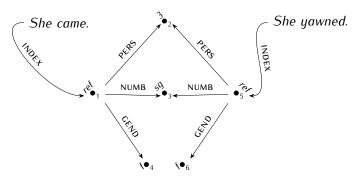
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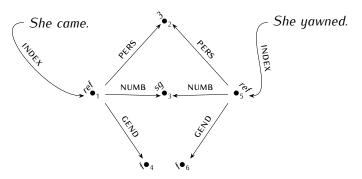
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Richter 2007: a typical HPSG grammar licenses as separate configurations, e.g., *phrases* with nonempty SLASH values.

Elements of SLASH are of sort local.

Various HPSG principles formulated as **constraints on** *signs* **constrain** *local* **configurations** within those *signs*.

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Höhle's problem 20000000 Exhaustive models

Extension of signature:

component/2

Extension of theory:

- UNIQUE U-SIGN CONDITION: $\forall \mathbb{1} \forall \mathbb{2}((\mathbb{1} \sim u_sign \land \mathbb{2} \sim u_sign) \rightarrow \mathbb{1} \approx \mathbb{2})$
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Exhaustivity implies that models contain all licensed configurations. But **not all configurations correspond to actual utterance tokens**. So there must be **non-actual** utterance **tokens**.

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 - we may separate the questions (Pullum and Scholz 2001):
 - Is a given configuration a model of a grammar?
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- no stranded structures: ditto (assuming we can say: utterance \rightarrow empty SLASH, etc.),
- no non-actual tokens:
 - we may separate the questions (Pullum and Scholz 2001):
 - Is a given configuration a model of a grammar?
 - What is the exact collection of models of a grammar?
 - we don't have to enumerate all possible (actual and non-actual) tokens.

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Solution: Rooted models of utterances

A **solution** to all of the above (see extended slides for RSRL technicalities):

- rooted models,
- of single utterances.

Solves all the problems above:

- no twin structures: everything is a component of a single utterance,
- no stranded structures: ditto (assuming we can say: utterance \rightarrow empty SLASH, etc.),
- no non-actual tokens:
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Solution:	Technicali	ties 1		

Assume the standard notion of **signature** (we ignore chains throughout):

Definition (signature)

 Σ is a signature iff Σ is a septuple $\langle S, \subseteq, S_{max}, A, F, R, Ar \rangle$, $\langle S, \sqsubseteq \rangle$ is a partial order, $S_{max} = \{ \sigma \in S \mid \text{for each } \sigma' \in S, \text{ if } \sigma' \sqsubseteq \sigma \text{ then } \sigma = \sigma' \},$ A is a set, *F* is a partial function from $S \times A$ to *S*, for each $\sigma_1 \in S$, for each $\sigma_2 \in S$, for each $\phi \in A$, if $F(\sigma_1, \phi)$ is defined and $\sigma_2 \sqsubseteq \sigma_1$ then $F(\sigma_2, \phi)$ is defined and $F(\sigma_2, \phi) \sqsubseteq F(\sigma_1, \phi)$, R is a finite set, and Ar is a total function from R to the positive integers.



We extend the notion of terms – \mathring{r} will refer to the root of the utterance:

Definition (terms)

For each signature $\Sigma = \langle S, \sqsubseteq, S_{max}, A, F, R, Ar \rangle$, the set of terms T^{Σ} is the smallest set such that $\mathring{r} \in T^{\Sigma}$, $: \in T^{\Sigma}$, for each $x \in V$, $x \in T^{\Sigma}$, for each $\phi \in A$ and each $\tau \in T^{\Sigma}$, $\tau \phi \in T^{\Sigma}$.



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Solution:	Technicaliti	es	3		

Standard definition of formulæ:

Definition (formulæ)

For each signature $\Sigma = \langle S, \sqsubseteq, S_{max}, A, F, R, Ar \rangle$, the set of formulæ D^{Σ} is the smallest set such that for each $\sigma \in S$, for each $\tau \in T^{\Sigma}$, $\tau \sim \sigma \in D^{\Sigma}$, for each $\tau_1, \tau_2 \in T^{\Sigma}$, $\tau_1 \approx \tau_2 \in D^{\Sigma}$, for each $\rho \in R$, for each $x_1, \ldots, x_{Ar(\rho)} \in V$, $\rho(x_1, \ldots, x_{Ar(\rho)}) \in D^{\Sigma}$, for each $x \in V$, for each $\delta \in D^{\Sigma}$, $\exists x \delta \in D^{\Sigma}$, (analogous for \forall) for each $\delta \in D^{\Sigma}$, $\neg \delta \in D^{\Sigma}$, for each $\delta_1, \delta_2 \in D^{\Sigma}$, and $(\delta_1 \land \delta_2) \in D^{\Sigma}$.

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Solution:	Technicali	ties	4		

Trivial extension of the definition of **free variables** to handle the term r:

Definition (free variables)

For each signature $\Sigma = \langle S, \sqsubseteq, S_{max}, A, F, R, Ar \rangle$, FV is the function from terms and formulæ to free variables in them: $FV(\dot{r}) = \{\},\$ $FV(:) = \{\},\$ for each $x \in V$, $FV(x) = \{x\}$, for each $\tau \in T^{\Sigma}$, for each $\phi \in A$, $FV(\tau \phi) = FV(\tau)$, for each $\tau \in T^{\Sigma}$, for each $\sigma \in S$, $FV(\tau \sim \sigma) = FV(\tau)$, for each $\tau_1, \tau_2 \in T^{\Sigma}$, $FV(\tau_1 \approx \tau_2) = FV(\tau_1) \cup FV(\tau_2)$, for each $\rho \in R$, for each $x_1, \ldots, x_{Ar(\rho)} \in V$, $FV(\rho(x_1,\ldots,x_{Ar(\rho)})) = \{x_1,\ldots,x_{Ar(\rho)}\},\$ for each $\delta \in D^{\Sigma}$, for each $x \in V$, $FV(\exists x \delta) = FV(\delta) \setminus \{x\}$. (also \forall) for each $\delta \in D^{\Sigma}$, $FV(\neg \delta) = FV(\delta)$, for each $\delta_1, \delta_2 \in D^{\Sigma}$, $FV((\delta_1 \wedge \delta_2)) = FV(\delta_1) \cup FV(\delta_2)$. $(also \lor, \rightarrow, \leftrightarrow)$

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Solution:	Technicali	ties	4		

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Solution:	Technicaliti	es	5		

Standard definition of **descriptions**:

Definition (descriptions)

For each signature Σ , the set of descriptions $D_0^{\Sigma} = \{\delta \in D^{\Sigma} | FV(\delta) = \{\}\}.$

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Solution:	Technical	ities 6		
Definition o	of interpretation	on now singles out one	element of the univ	erse:
Definition (interpretation)		
For each si	gnature $\Sigma = \langle$	$(S, \sqsubseteq, S_{max}, A, F, R, Ar)$), $I=\langleU,r,S,A,R angle$ is	s an
Σ interprete	ation iff			
U is a set,				
$r \in U$,				
S is a total	function from	U to S_{max} ,		
A is a total	function from	A to the set of partial	functions from ${\sf U}$ to	U,
for each $\phi \in$	A and each	$u \in U$		
if $A(\phi)($	(u) is defined			
then F($S(u),\phi)$ is de	fined, and $S(A(\phi)(u))$!	$\equiv F(S(u), \phi)$, and	
for each $\phi \in$	A and each	$u \in U$,		
if $F(S(\iota$	$\mu),\phi)$ is define	ed then $A(\phi)(u)$ is defi	ned,	
R is a total	function from	R to the power set of	0	
for each	$\rho \in R, R(\rho) \subseteq$	$\equiv U^{Ar(\rho)}.$	$n \in \mathbb{N}$	

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Solution	: Technical	ities 6		
Definition	of interpretation	on now singles out on	e element of the unive	erse:
Definition	(interpretation))		
For each s	signature $\Sigma = \langle$	$S, \subseteq, S_{max}, A, F, R, A$	r \rangle , I = $\langle U, r, S, A, R \rangle$ is	an
Σ interpre	tation iff		, , , , , , , , , , , , , , , , , , ,	
U is a set,				
$r \in U$,				
S is a tota	l function from	U to S_{max} ,		
A is a tota	al function from	A to the set of partia	l functions from U to	U,
for each ϕ	$\in A$ and each	$u \in U$		
	(u) is defined			
then F	$F(S(u),\phi)$ is defined	fined, and $S(A(\phi)(u))$	$\sqsubseteq F(S(u), \phi)$, and	
'	$\in A$ and each			
if $F(S)$	$(u),\phi)$ is define	ed then $A(\phi)(u)$ is def	ined,	
R is a tota	al function from	R to the power set of	0	
for eac	$ch \ \rho \in R, \ R(\rho) \subseteq$	$\equiv U^{Ar(\rho)}.$	n∈ℕ	

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Solution:	Technicali	ties	7		

Standard definition of variable assignments:

Definition (variable assignments)

For each signature Σ , for each Σ interpretation $I = \langle U, r, S, A, R \rangle$, $G_I = U^V$ is the set of variable assignments in I.

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Solution:	Technicaliti	es 8		

Standard **term interpretation**, extended so that the interpretation of \mathring{r} is r:

Definition (term interpretation)

For each signature $\Sigma = \langle S, \sqsubseteq, S_{max}, A, F, R, Ar \rangle$, for each Σ interpretation $I = \langle U, r, S, A, R \rangle$, for each $g \in G_I$, the term interpretation T_I^g is the total function from T^Σ to the set of partial functions from U to U such that for each $u \in U$, $T_I^g(\hat{r})(u)$ is defined and $T_I^g(\hat{r})(u) = r$, $T_I^g(:)(u)$ is defined and $T_I^g(:)(u) = u$, for each $x \in V$, $T_I^g(x)(u)$ is defined and $T_I^g(x)(u) = g(x)$, for each $\tau \in T^\Sigma$, for each $\phi \in A$, $T_I^g(\tau \phi)(u)$ is defined iff $T_I^g(\tau)(u)$ is defined and $A(\phi)(T_I^g(\tau)(u))$ is defined, and

if $T_{I}^{g}(\tau\phi)(u)$ is defined then $T_{I}^{g}(\tau\phi)(u) = A(\phi)(T_{I}^{g}(\tau)(u))$.

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Solution:	Technicalit	ies	8		

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 $\begin{aligned} \mathsf{T}_{\mathsf{I}}^{g}(\mathring{r})(u) & \text{ is defined and } \mathsf{T}_{\mathsf{I}}^{g}(\mathring{r})(u) = \mathsf{r}, \\ \mathsf{T}_{\mathsf{I}}^{g}(:)(u) & \text{ is defined and } \mathsf{T}_{\mathsf{I}}^{g}(:)(u) = u, \\ \text{for each } x \in V, \ \mathsf{T}_{\mathsf{I}}^{g}(x)(u) & \text{ is defined and } \mathsf{T}_{\mathsf{I}}^{g}(x)(u) = g(x), \\ \text{for each } \tau \in T^{\Sigma}, \text{ for each } \phi \in A, \\ \mathsf{T}_{\mathsf{I}}^{g}(\tau\phi)(u) & \text{ is defined iff } \mathsf{T}_{\mathsf{I}}^{g}(\tau)(u) & \text{ is defined } \\ & \text{ and } \mathsf{A}(\phi)(\mathsf{T}_{\mathsf{I}}^{g}(\tau)(u)) & \text{ is defined, and} \\ & \text{ if } \mathsf{T}_{\mathsf{I}}^{g}(\tau\phi)(u) & \text{ is defined then } \mathsf{T}_{\mathsf{I}}^{g}(\tau\phi)(u) = \mathsf{A}(\phi)(\mathsf{T}_{\mathsf{I}}^{g}(\tau)(u)). \end{aligned}$



Formula denotation – now quantification is over the whole universe (i.e., over all the components of the utterance, similarly to Richter 2007):

Definition (formula denotation)

. . .

For each signature $\Sigma = \langle S, \sqsubseteq, S_{max}, A, F, R, Ar \rangle$, for each Σ interpretation $I = \langle U, r, S, A, R \rangle$, for each $g \in G_I$, the formula denotation function D_1^g is the total function from D^{Σ} to the power set of U such that for each $\tau \in T^{\Sigma}$, for each $\sigma \in S$, $\mathsf{D}^{g}_{\mathsf{I}}(\tau \sim \sigma) = \left\{ u \in \mathsf{U} \middle| \begin{array}{c} \mathsf{T}^{g}_{\mathsf{I}}(\tau)(u) \text{ is defined, and} \\ \mathsf{S}\left(\mathsf{T}^{g}_{\mathsf{I}}(\tau)(u)\right) \sqsubseteq \sigma \end{array} \right\},$ for each $\tau_1, \tau_2 \in T^{\Sigma}$, $\mathsf{D}^{g}_{\mathsf{I}}(\tau_{1} \approx \tau_{2}) = \left\{ u \in \mathsf{U} \middle| \begin{array}{c} \mathsf{T}^{g}_{\mathsf{I}}(\tau_{1})(u) \text{ is defined,} \\ \mathsf{T}^{g}_{\mathsf{I}}(\tau_{2})(u) \text{ is defined, and} \\ \mathsf{T}^{g}_{\mathsf{I}}(\tau_{1})(u) = \mathsf{T}^{g}_{\mathsf{I}}(\tau_{2})(u) \end{array} \right\},$ for each $\rho \in R$, for each $x_1, \ldots, x_{Ar(\rho)} \in V$, $\mathsf{D}^{g}_{\mathsf{L}}\left(\rho(x_{1},\ldots,x_{\mathsf{Ar}(\rho)})\right) = \left\{ u \in \mathsf{U} \mid \left\langle g(x_{1}),\ldots,g(x_{\mathsf{Ar}(\rho)})\right\rangle \in \mathsf{R}(\rho) \right\},\$

Introduction O	Höhle's problem 00000000	Exhaustive models	Second-order extension	Coda O
Solution	Technical	ities 10		
Definition	(formula denot	ation contd.)		
	h $x \in V$, for ea			
$D^{g}_{I}(\exists$	$(x\delta) = \begin{cases} u \in U \end{cases}$	$ \left \begin{array}{c} \text{for some } u' \in U \\ u \in D_{I}^{g[x \mapsto u']}(\delta) \end{array} \right\rangle' $		
	h $x \in V$, for ea			
$D_{I}^{g}\left(\forall$	$(x\delta) = \begin{cases} u \in U \end{cases}$	$\left \begin{array}{l} \text{for each } u' \in U \\ u \in D_{I}^{g[x \mapsto u']}(\delta) \end{array} \right\rangle,$		
	I \	$\neg \delta) = U \backslash D^{g}_{I}(\delta),$		
	· · · · · · · · · · · · · · · · · · ·	$D^{g}_{L}\left(\left(\delta_{1}\wedge\delta_{2}\right)\right)=D^{g}_{L}$		
		$D^{g}_{L}\left(\left(\delta_{1} \lor \delta_{2}\right)\right) = D^{g}_{L}$		
	_	$D^{g}_{I}\left(\left(\delta_{1}\to\delta_{2}\right)\right)=\left(U^{g}_{1}\right)$	$\mathbb{U} \setminus D^{g}_{I}(\delta_{1}) \big) \cup D^{g}_{I}(\delta_{2})$, an	d
	h $\delta_1, \delta_2 \in D^{\Sigma}$,			
D^{g}_{I} ((δ	$(\iota \leftrightarrow \delta_2)) = ((\iota$	$J\backslashD^{g}_{I}(\delta_{1}))\cap(U\backslashD^{g}_{I}($	$(\delta_2))) \cup (D^{g}_{I}(\delta_1) \cap D^{g}_{I}(\delta_2)))$	i ₂)).

Introduction O	Höhle's problem 00000000	Exhaustive models	Second-order extension	Coda O
Solution	: Technical	ities 10		
Definition	(formula denot	ation contd.)		
	h $x \in V$, for ea	· · · · · · · · · · · · · · · · · · ·		
$D^{g}_{I}(\exists$	$(x\delta) = \begin{cases} u \in U \end{cases}$	$\begin{cases} \text{for some } u' \in U \\ u \in D_{I}^{g[x \mapsto u']}(\delta) \end{cases}, \end{cases}$		
	$f(x) \in V$, for each $x \in V$, for each X , f			
$D_{I}^{g}\left(\forall$	$(x\delta) = \begin{cases} u \in U \\ \end{bmatrix}$	$\begin{cases} \text{for each } u' \in U \\ u \in D_{I}^{g[x \mapsto u']}(\delta) \end{cases}, \end{cases}$		
		$\neg \delta) = U \backslash D^{g}_{I}(\delta),$		
	· · · · · · · · · · · · · · · · · · ·	$D^{g}_{I}\left(\left(\delta_{1} \wedge \delta_{2}\right)\right) = D^{g}_{I}($		
		$D^{g}_{L}\left(\left(\delta_{1} \lor \delta_{2}\right)\right) = D^{g}_{L}\left(\left(\delta_{1} \lor \delta_{2}\right)\right) = D^{g}_{L}\left(\left(\delta_{1} \lor \delta_{2}\right)\right)$		
	-	$D^{g}_{I}\left(\left(\delta_{1}\to\delta_{2}\right)\right)=\left(U\right.$	$\setminus D^{g}_{I}(\delta_{1}) ig) \cup D^{g}_{I}(\delta_{2})$, an	d
	h $\delta_1, \delta_2 \in D^{\Sigma}$,			
$D^{g}_{I}\left(\left(\delta\right)$	$(\iota \leftrightarrow \delta_2)) = ((\iota$	$J\backslashD^{g}_{I}(\delta_{1}))\cap(U\backslashD^{g}_{I}(\delta_{1}))$	$(\mathfrak{H}_{2}))) \cup (D_{I}^{g}(\delta_{1}) \cap D_{I}^{g}(\delta_{1})))$	2)).

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Solution	Technicali	ities 11		

Standard definition of **description denotation**:

Definition (description denotation)

For each signature Σ , for each Σ interpretation $I = \langle U, r, S, A, R \rangle$, the description denotation function D_I is the total function from D_0^{Σ} to the power set of U such that $D_I(\delta) = \{ u \in U \mid \text{for each } g \in G_I, u \in D_I^g(\delta) \}$.

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Solution:	Technicali	ties	12		

We want to make sure that r is the root...

• $\forall \texttt{1} \texttt{ component}(\texttt{1}, \mathring{r})$ i.e.: $\exists \texttt{0} (\texttt{0} \approx \mathring{r} \land \forall \texttt{1} \texttt{ component}(\texttt{1}, \texttt{0}))$

- ${r}$ synsem nonlocal slash \sim eset
- ${r}$ synsem local cat val subj ~ elist \land ${r}$ synsem local cat val comps ~ elist
- etc.

Introduction O	Höhle's problem 00000000	Exhaustive	e models >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>	Second-order extension	Coda O
Solution:	Technicali	ties	12		

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- etc.

Rooted models - not an original idea:

- discussed **in modal logic** (*rooted* or *point generated* models; Blackburn *et al.* 2010:56),
- these are exactly **Pollard's (1999: §6) singly generated** models.

Here, we **additionally**:

- require that such rooted models correspond to utterances,
- allow terms to **refer to the root** directly.

- utterance tokens and their parts (King 1999),
- **unique mathematical objects representing utterance types** (Pollard and Sag 1994, Pollard 1999).

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Yatabe 2004 deals with unlike category coordination, e.g. (Bayer 1996):

- We emphasized [[Mr. Colson's many qualifications]_{NP} and [that he had worked at the White House]_{CP}].
- We emphasized [[that Mr. Colson had worked at the White House]_{CP} and [his many other qualifications]_{NP}].

His lexical entry for emphasized: Image: Phon (emphasized) Image: Subject (moun control co

The key idea is the use of the **distributive functor**, *c*:

•
$$1: c(\alpha) \equiv 1: \alpha \lor$$

 $(1: [\operatorname{ARGS} \langle \overline{a_1}, \dots, \overline{a_n} \rangle] \land \overline{a_1}: \alpha \land \dots \land \overline{a_n}: \alpha)$



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The key idea is the use of the **distributive functor,** *c*:

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$$1: c(\alpha) \equiv 1: \alpha \lor$$

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Introduction	Höhle's problem	Exhaustive models	Second-order extension	Coda
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Introduction O	Höhle's problem 00000000	Exhaustive models	Second-order extension	Coda O	

Unlike category coordination in HPSG

Unlike category coordination (UCC) in HPSG:

- linearisation-based approaches to UCC were popular in HPSG in 2000s (e.g., Crysmann 2003, Beavers and Sag 2004, Chaves 2006, 2007, 2008).
- many cases of UCC not amenable to such elliptical approaches (Levine 2011, Dalrymple 2017, Patejuk and Przepiórkowski 2021),
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Coda O

Reformulation in second-order RSRL

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• $\begin{bmatrix} \mathsf{PHON} & \langle emphasized \rangle \\ \dots & \mathsf{VALENCE} & \begin{bmatrix} \mathsf{SUBJ} & \langle [\dots, \mathsf{HEAD} \ \fbox{l}] \rangle \\ \mathsf{COMPS} & \langle [\dots, \mathsf{HEAD} \ \fbox{l}] \rangle \end{bmatrix} \end{bmatrix} \xrightarrow{\begin{subarray}{c} \alpha_1 \approx (: \sim \textit{noun} \land : \mathsf{CASE} \sim \textit{nom}) \\ \land & \alpha_2 \approx (: \sim \textit{noun} \lor : \sim \textit{comp}) \\ \land & \alpha_2 \approx (: \sim \textit{noun} \lor : \sim \textit{comp}) \\ \land & \mathsf{c}(\fbox{l}, \alpha_1) \land \mathsf{c}(\fbox{l}, \alpha_2) \end{bmatrix}$

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What would it require?

• signatures **specify types of arguments of relation symbols** (each either *e* or *et*);

1

- interpretations of relation symbols:
 - sets of tuples (as before), but
 - each element of a tuple is an object or a set of objects,
 - depending on the type specified in the signature;
- the set of variables, VAR, is the disjoint sum of:
 - \mathcal{VAR}_e (first-order variables) and
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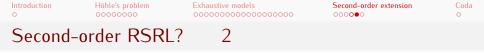
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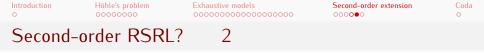
Introduction O	Höhle's problem 00000000	Exhaustive models	Second-order extension	Coda O
Second-o	rder RSRL?	2		

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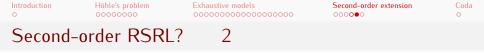
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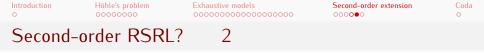
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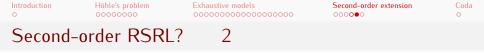
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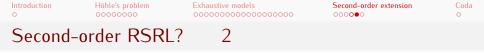
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Introduction O	Höhle's problem 00000000	Exhaustive models	Second-order extension	Coda O
Open q	uestions			

- is it worth it?
- a less elegant alternative:
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Solutions

- General Identity Principle (GIP; Sailer 2003: 116),
- modified to exempt some structures (e.g., of sort ref) from its scope;
- **exhaustive models** (twin structures, stranded structures, ontological problems):
 - modification of RSRL,
 - each model corresponds to an utterance,
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 - each model corresponds to an utterance,
 - and has a distinguished element the root;
- unlike category coordination (Yatabe 2004):
 - extension of RSRL,
 - values of variables arguments of relations may be objects or properties of objects,
 - i.e., second-order RSRL.

Coda •

Instead of conclusion

Solutions:

- General Identity Principle (GIP; Sailer 2003: 116),
- modified to exempt some structures (e.g., of sort ref) from its scope;
- **exhaustive models** (twin structures, stranded structures, ontological problems):
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Solutions:

• "Höhle's problem":

- General Identity Principle (GIP; Sailer 2003: 116),
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 - and has a distinguished element the **root**;
- unlike category coordination (Yatabe 2004):
 - extension of RSRL,
 - values of variables arguments of relations may be objects or properties of objects,
 - i.e., second-order RSRL.

Thank you for your attention!

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